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# MOMENT THEORY OF ELECTROMAGNETIC EFFECTS IN ANISOTROPIC SOLIDS*

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A moment (polar) theory of deformable solids is constructed for anisotropic media such as polarizable piezoelectric ceramics. The linear theory is considered in detail and an explanation of the non-linear change in the electric field inside a polarized piezoelectric material (the Mead effect) is given. The classical theory of electromagnetic effects in solids does not enable certain observed effects to be described (for example, the Mead effect /1/). Attempts to eliminate this drawback of classical theory /2, 3/ rest on the introduction of the polarization gradient into the enthalpy as a parameter of the process. Models of complex media which takes into account the internal mechanical and electromagnetic moments have been constructed in electrodynamics (for example /4, 5/) when electromagnetic fields interact with the medium. Below, a solution of the problem is given and an example of a natural description of the Mead effect is presented.

Suppose $x^i (i = 1, 2, 3)$ is a Lagrange system of coordinates frozen into a medium which occupies a volume $V$ with a boundary $S$. The vector $\mathbf{r}(x^i, t)$, defines the position of a point of this medium with respect to a fixed inertial system $y^i$, where $t$ is the time. The vector $\mathbf{r}^* = \mathbf{r} + \mathbf{u}(x^i, t)$ defines the position of material points of the medium after strain, where $\mathbf{u}$ is the displacement vector. Further constructions which are carried out have the purpose of

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describing the behaviour of piezoelectric ceramic media which are brittle, and naturally, cannot be subjected to appreciable strain or bending. For this reason the further constructions are carried out in a geometrically linear formulation. For small displacements, Green's strain tensor has covariant components

$$\mathbf{E} = \varepsilon_{ik}\mathbf{r}^i\mathbf{r}^k, \quad 2\varepsilon_{ik} = \nabla_i u_k + \nabla_k u_i = \mathbf{r}_i\cdot\partial\mathbf{u}/\partial x^k + \mathbf{r}_k\cdot\partial\mathbf{u}/\partial x^i$$

while the Cauchy stress tensor has contravariant components $\sigma^{ik}$. The stress vector on an area with unit vector $\mathbf{n} = n^i\mathbf{r}_i$ is

$$\mathbf{P}_n = \sigma^{ik}\mathbf{r}_k n_i; \quad \Sigma = \sigma^{ik}\mathbf{r}_i\mathbf{r}_k \not\equiv \sigma^{ik}\mathbf{r}_k\mathbf{r}_i \tag{1}$$

while the vector of the internal moment on the same area is

$$\boldsymbol{\mu}_n = \mu^{ik} n_i \mathbf{r}_k; \quad \mathbf{M} = \mu^{ik}\mathbf{r}_i\mathbf{r}_k \not\equiv \mu^{ik}\mathbf{r}_k\mathbf{r}_i \tag{2}$$

where $\mu^{ik}$ are the components of the moment tensor. It follows from the equations of equilibrium for the forces and moments for an elementary volume that

$$\nabla_i\sigma^{ik} + Q^k = 0, \quad \nabla_i\mu^{ij} + c^{ikj}\sigma_{ik} + \mu^i = 0 \tag{3}$$

where $Q^i\mathbf{r}_i$, $\mu^i\mathbf{r}_i$ are vectors referred to unit volume of the mass forces and moments. It follows from (3) that in general $\sigma^{ik} \neq \sigma^{ki}$.

We will use the condition that for an arbitrary volume of the body the sum of the second-order moments $[\mathbf{r}\times[\mathbf{r}\times\mathbf{F}]]$ with respect to an internal point is in equilibrium. We then obtain the equality $\mu^{ik} = \mu^{ki}$, which will henceforth be assumed to be satisfied.

The sum of the work done by the external forces and moments

$$\delta A = \int \mathbf{Q}\cdot\delta\mathbf{u}\, dV + \int \mathbf{P}_n\cdot\delta\mathbf{u}\, dS + \int \boldsymbol{\mu}\cdot\delta\boldsymbol{\omega}\, dV + \int \boldsymbol{\mu}_n\cdot\delta\boldsymbol{\omega}\, dS$$

where $\omega$ is the rotation vector, and, taking expressions (1) and (2) into account, this can be converted to the form

$$\delta A = \int t^{ik}\delta\varepsilon_{ik}\, dV + \int \mu^{ik}\delta\xi_{ik}\, dV + \int \nu^i\delta\gamma_i\, dV$$

Here

$$2t^{ik} = \sigma^{ik} + \sigma^{ki}, \quad 2\nu^j = c^{jik}l_{ik}, \quad 2l^{ik} = \sigma^{ik} - \sigma^{ki}$$
$$2\varepsilon_{ik} = \nabla_i u_k + \nabla_k u_i, \quad 2\xi_{ik} = \nabla_i\omega_k + \nabla_k\omega_i, \quad 2\eta_{ik} = \nabla_i u_k - \nabla_k u_i$$
$$2\gamma^j = c^{jik}\eta_{ik} - \omega^j$$

In this notation, Eqs.(3) take the form

$$\nabla_i t^{ik} - c^{ikj}\nabla_i\nabla_m\mu^{m}_{\cdot j} = c^{ikj}\nabla_i\mu_j - Q^k \tag{4}$$
$$\nu^j = -\nabla_i\mu^{ij} - \mu^j \tag{5}$$

Above and everywhere henceforth integration is carried out over the volume $V$ and the surface $S$.

The increment of energy of the electromagnetic field in the volume $V$ and the amount of heat dissipated due to the fluxes $\mathbf{q}_h$ and $\mathbf{q}_e$ of the magnetic field $\mathbf{H}$ and the electric field $\mathbf{E}$ can be expressed as follows ($\mathbf{B}$ and $\mathbf{D}$ are the magnetic and electric induction and $\mathbf{I}$ is the current strength):

$$\delta W = \int q_h{}^i\delta E_i dS + \int q_e^i\delta H_i\, dS = \delta\int(\mathbf{E}\cdot\mathbf{D}^{\cdot} + \mathbf{H}\cdot\mathbf{B}^{\cdot})\, dV + \delta\int \mathbf{E}\cdot\mathbf{I}\, dV \tag{6}$$

where the right-hand side is taken as the energy by definition /6/.

By adding to the middle and right-hand sides the terms

$$-\int n_j c^{ikj} H_i\delta E_k dS + \int n_j c^{jik} E_i\delta H_k dS$$

we obtain

$$\int (q_h{}^k - n_j c^{jik} H_i)\,\delta E_k dS + \int (q_e{}^k + n_j c^{jik} E_i)\,\delta H_k dS =$$
$$\int (D^{\cdot k} + I^k - c^{jik}\nabla_j H_i)\,\delta E_k dV + \int (B^{\cdot k} + c^{jik}\nabla_j E_i)\,\delta H_k dV \tag{7}$$

When Maxwell's equations are satisfied the right-hand side of the last equation is zero and, consequently, for arbitrary variations $\delta E_k$ and $\delta H_k$, the boundary conditions for the fluxes $\mathbf{q}_h$ and $\mathbf{q}_e$ follow from (7). Hence, relations (6) and (7) are analogues of Lagrange's variational equation, written for an electromagnetic field.

Including in our considerations the inflow of heat due to the flux vector of its **q** through the surface $S$ and due to internal sources of intensity $r$, we obtain the following equation for the increment of the internal energy of the medium:

$$dU = \delta A + \delta W + \left(\int r\,dV\right)dt - \left(\int \mathbf{q}\cdot\mathbf{n}\,dS\right)dt \tag{8}$$

The total amount of heat absorbed by the body in a time $dt$ is

$$\delta Q = \left(-\int \mathbf{q}\cdot\mathbf{n}\,dS + \int r\,dV + \int \mathbf{E}\cdot\mathbf{I}\,dV + \int W^{\cdot} dV\right)dt$$

where $W^{\cdot}$ is the rate of generation of heat due to conversion of mechanical energy and the energy of the interaction of the electromagnetic and mechanical fields into heat. Using Gauss's theorem for the rate $s^{\cdot}$ of increase of entropy per unit volume, we obtain ($T$ is the absolute temperature)

$$Ts^{\cdot} = -\nabla_i q^i + r + \sigma, \quad \sigma = E\cdot I + W^{\cdot} \tag{9}$$

In view of the fact that $q_i \nabla^i T \leqslant 0$ and $\dot{\sigma} \geqslant 0$ for irreversible processes, the Clausius-Duhem inequality

$$s^{\cdot} + \nabla_i (q^{\cdot}/T) - r/T \geqslant 0$$

follows from (9).

Introducing the free energy $F = u - Ts$ we obtain from (8)

$$F^{\cdot} = t^{ik}\varepsilon_{ik}^{\cdot} + \mu^{ik}\xi_{ik}^{\cdot} + \nu^i\gamma_i^{\cdot} + E^i D_i^{\cdot} + H^i B_i^{\cdot} - sT^{\cdot} - W^{\cdot}$$
$$W^{\cdot} = \sum W_\chi{}^A \chi_A^{\cdot}$$

where $\chi_A$ are additional parameters of the process with a generalized tensor index $A$, and $W_\chi{}^A$ are generalized forces corresponding to them.

Further constructions depend on the choice of the parameters $\chi_A$ and the functions $W_\chi{}^A$. If, for example $\varepsilon_{ij(n)}, \xi_{ij(n)}, \gamma_{i(n)}$ are the irreversible components of the strains, in which $t_{ik}, \mu_{ik}, \nu_i$ perform work which is dissipated in the form of heat with powers

$$t^{ij}\dot{\varepsilon}_{ij(n)} \geqslant 0, \quad \mu^{ij}\dot{\xi}_{ij(n)} \geqslant 0, \quad \nu^i\dot{\gamma}_{i(n)} \geqslant 0$$

then, using the notation

$$\dot{\varepsilon}_{ij(e)} = \varepsilon_{ij}^{\cdot} - \dot{\varepsilon}_{ij(n)}, \quad \dot{\xi}_{ij(e)} = \xi_{ij}^{\cdot} - \dot{\xi}_{ij(n)}, \quad \dot{\gamma}_{i(e)} = \gamma_i^{\cdot} - \dot{\gamma}_{i(n)} \tag{10}$$

we obtain from the expression for $F^{\cdot}$

$$t^{ij} = \partial F/\partial \varepsilon_{ij(e)}, \quad \mu^{ij} = \partial F/\partial \xi_{ij(e)}, \quad \nu^j = \partial F/\partial \gamma_{j(e)}$$
$$E^k = \partial F/\partial D_k, \quad H^k = \partial F/\partial B_k, \quad s = -\partial F/\partial T \tag{11}$$

It is obvious that $\dot{\varepsilon}_{ij(e)}, \dot{\xi}_{ij(e)}, \dot{\gamma}_{i(e)}$ is the rate of increase of elastic (locally irreversible) strains, while the first three groups of Eqs.(11) represent generalizations of Green's formulas, which are well-known in the theory of elasticity, to the case of inelastic strains. Similar generalizations are possible for the groups of formulas (11) for $E_k$ and $H_k$, if irreversible parts of the increments of the quantities $D_i$ and $B_i$ can exist. For $\chi_A^{\cdot}$ we must introduce evolution equations /7/. Replacing $s^{\cdot}$ in (9) by its expression

$$s^{\cdot} = -d\,(\partial F/\partial T)/dt$$

which follows from (11), we obtain an equation for the heat flux.

We will further assume that $I = 0$, which occurs in dielectrics, and we will assume that the deformation process is reversible over the range of variation of the parameters of the process considered.

For crystals of the class (6mm) /8/ with one axis of mechanical and electrical symmetry, for which we take the $x_3$ axis of a Cartesian system of coordinates $x_1x_2x_3$, the number of fundamental parameters of the process will include the scalars $\varepsilon_{33}, \xi_{33}, D_3, B_3, \gamma_3, T$, the vectors $\varepsilon_{\alpha 3}, \xi_{\alpha 3}, D_\alpha, B_\alpha, \gamma_\alpha$, and the tensors $\varepsilon_{\alpha\beta}, \xi_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$.

We will introduce the following limits. Suppose the process is isothermal and the effect of the magnetic field is small. This is the case in dielectrics. Moreover, we will assume that $\gamma_3 \simeq 0$, $\gamma_\alpha \simeq 0$ (similar to Kirchhoff's hypotheses in the theory of thin plates). In this case, the quantities $v^i$ are found from conditions (5) after solving Eq.(4), which reduces to three equations with three required functions $u_i(x_k, t)$. In addition we will assume that $W^* = 0$. With these limitations, the arguments of the function $F$ will be

$$\varepsilon_{33}, \xi_{33}, D_3; \quad \varepsilon_{\alpha 3}\varepsilon^{\alpha 3}, \quad \xi_{\alpha 3}\xi^{\alpha 3}, \quad D_\alpha D^\alpha; \quad \varepsilon^{\alpha}_{\alpha \cdot}, \quad \varepsilon_{\alpha\beta}\varepsilon^{\alpha\beta}, \quad \varepsilon_{3\alpha}\varepsilon^{\alpha\beta}\varepsilon_{\beta 3}; \quad \xi^{\alpha}_{\alpha \cdot}, \quad \xi_{\alpha\beta}\xi^{\alpha\beta}, \quad \xi_{\alpha 3}\xi^{\alpha\beta}\xi_{\beta 3} \tag{12}$$

and the mixed (combined) invariants will be

$$\varepsilon_{3\alpha}D^\alpha, \quad \xi_{3\alpha}D^\alpha, \quad \xi_{\alpha 3}\varepsilon^{\alpha 3}, \quad \xi_{\alpha\beta}\varepsilon^{\alpha\beta}, \quad \varepsilon_{3\alpha}\varepsilon^{\alpha\beta}\xi_{\beta 3}, \quad \xi_{3\alpha}\xi^{\alpha\beta}\varepsilon_{\beta 3}, \ldots \tag{13}$$

which represent the mutual orientation of the tensors and vectors occurring in them. Of the mixed invariants (13) in the number of arguments of the function $F$ we can only include those of them that satisfy the condition that all the set of arguments of the number (12) and (13) are independent. For example, the set of components $\varepsilon_{ik}$, $\xi_{ik}$, $D_i$ contains 15 parameters. Hence, to the twelve basic arguments (12) we can additionally add not more than three invariants from (13), whereas the remaining ones will be numerically dependent on the previously chosen independent fifteen.

The defining relations (11) and the system of arguments (12) and (13) introduced, which can be generalized in a natural way to crystals with lower symmetry, enable us to construct fairly general forms of relations between the fields, taking into account the physical non-linearity and the effect of the temperature.

In order to demonstrate the possibility of describing the Mead effect within the framework of the relations constructed, we will introduce a number of simplifying assumptions. Suppose the fields $\varepsilon_{ik}$ and $\xi_{ik}$ are weakly coupled. Of the invariants (13) we need then only retain the first two. Taking this into account and retaining only the second powers in the expansion for $F$ in powers of the main parameters, and taking into account the unstressed nature of the initial state, we obtain the defining relations (11) in the form

$$\begin{gathered} t_{11} = c_{11}\varepsilon_{11} + c_{12}\varepsilon_{22} + c_{13}\varepsilon_{33} - e_{31}D_3, \quad t_{22} = c_{12}\varepsilon_{11} + c_{11}\varepsilon_{22} + c_{13}\varepsilon_{33} - \\ - e_{31}D_3, \quad t_{33} = c_{13}\varepsilon_{11} + c_{13}\varepsilon_{22} + c_{33}\varepsilon_{33} - e_{33}D_3 \\ t_{13} = 2c_{44}\varepsilon_{13} - e_{15}D_1, \quad t_{23} = 2c_{44}\varepsilon_{23} - e_{15}D_2 \\ t_{12} = 2c_{66}\varepsilon_{12} = (c_{11} - c_{22})\varepsilon_{12} \\ E_1 = -e_{15}\varepsilon_{13} - d_{15}\xi_{13} + \lambda_1 D_1, \quad E_2 = -e_{15}\varepsilon_{23} - d_{15}\xi_{23} + \lambda_1 D_1, \\ E_3 = -e_{31}\varepsilon_{11} - e_{31}\varepsilon_{22} - e_{33}\varepsilon_{33} - f_{31}(\xi_{11} + \xi_{22}) - f_{33}\xi_{33} + \lambda_3 D_3 \end{gathered} \tag{14}$$

(the relations for $\mu_{ik}$ are obtained from the relations for $t_{ik}$ by replacing $\varepsilon_{ik}$, $c_{ik}$, $e_{ik}$ by $\xi_{ik}$, $d_{ik}$, $f_{ik}$ respectively).

The relations for $t_{ik}$ and $E_i$, ignoring terms with $\xi_{ik}$, repeat those usually employed (/8-11/ etc.).

For the kinematic characteristics we have (the unwritten relations are obtained by cyclic permutation of the indices 1, 2, 3)

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \quad 2\varepsilon_{12} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}$$

$$\xi_{11} = \frac{\partial}{\partial x_1}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right), \quad 2\xi_{12} = \frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right)$$

We will consider a piezoelectric ceramic element of thickness $2h$ along the $x_1$ axis, when the element is polarized in the direction of the $x_3$ axis. An external electrostatic field with vector $\mathbf{E} = E_1\mathbf{e}_1$ acts on the specimen. Then, over the whole crystal $t_{11} = t_{22} = t_{33} = 0$, $D_3 = D_2 = 0$ and, consequently, $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = 0$. Since only the additional moments $\mu_2 = \alpha E_1$ operate due to the action of the field $E_1$ on the dipoles oriented along the $x_3$ axis, we have $\varepsilon_{12} = 0$, $\varepsilon_{23} = 0$, $2\varepsilon_{13} = \partial u_3/\partial x_1$ when $u_1 = 0$, $u_3 = u_3(x_1)$. In addition, $\xi_{11} = \xi_{22} = \xi_{33} = 0$, $2\xi_{12} = -\partial^2 u_3/\partial x_1{}^2$. Together with this we have

$$t_{13} = c_{44}dw/dx - e_{15}D_1, \quad \mu_{13} = -D_1 d_{15}, \quad \mu_{23} = 0$$
$$\mu_{12} = -{}^1/{}_2\,(d_{11} - d_{22})d^2w/dx^2 \quad (w = u_3, \ x = x_1)$$

Eq.(4) for $k = 3$ can be written in the following form:

$$2d^2w/dx^2 - (d_{11} - d_{22})d^4w/dx^4 + \alpha dE_1/dx = 0 \tag{15}$$

Moreover, by (14) we have

$$2E_1 = -e_{15}dw/dx + 2\lambda_1 D_1 \tag{16}$$

In the polarized crystal considered, due to the action of the external electric field $E_{1e}$

there will be uncompensated charges at the boundaries $x = \pm h$, whereas inside the charge density will be zero. Hence, in view of Maxwell's equation for the divergence of the induction it follows that $D_{1d} = \mathrm{const}$ inside the dielectric when $-h < x < h$. Eliminating the electric field $E_{1d}$ in the dielectric from Eqs.(15) and (16) we obtain a uniform equation, the solution of which is

$$w(x) = A\,\mathrm{sh}\,\omega x + B\,\mathrm{ch}\,\omega x + Cx + D$$
$$\omega^2 = (d_{11} - d_{22})/(2c_{11} - \alpha e_{15})$$

In view of the conditions of the problem, the function $w(x)$ is skew symmetric with respect to $x$ and, consequently, $B = D = 0$. At the boundaries $x = \pm h$ we have $t_{13} = 0$, and elimination of the rigid rotation by the condition $dw/dx = 0$ when $x = 0$ gives

$$C = -\omega A, \quad A = e_{15}E_1/[c_{44}\omega(\mathrm{ch}\,\omega h - 1)] \tag{17}$$

Since, on passing through the boundary of the dielectric, the induction $D_1$ should retain its value, while in a vacuum $D_{1e} = E_{1e}$, and whereas in a dielectric, together with (16), we must have $D_{1d} = E_{1d} + 4\pi P$, where $P$ is the polarization, which in the case considered, according to (16), is represented by the term with $dw/dx$, we have $\lambda_1 = 1$. Hence, in the dielectric the electric field is given by the equation

$$E_{1d} = E_1 - e_{15}A\omega\,(\mathrm{ch}\,\omega x - 1)/2$$

where $A$ is given by the second equation of (17). Hence, the electric field inside the dielectric is non-linearly variable, which leads to the Mead effect. When $E_1 = 0$, $E_2 = 0$, $E_3 \neq 0$ this efect should not be observed in the type of crystals considered.

When constructing a non-linear theory, in general we must include the invariants (12) and (13) in the number of arguments of the function $F$. By taking into account the non-linearity connected with the dissipation of energy, we ensure an appropriate form of the function or the functional $W^*$, in which it is also possible to take into account the dissipation of the energy of the electromagnetic field itself and its interaction with the medium using models such as Maxwell's, Voigt's Boltzmann-Volterra etc.

To describe the Mead effect in crystals which are unpolarized from the beginning, but are polarized due to an external electric field, we must introduce the tensor $\nabla_i D_j$ as the argument of the function $F$ in (11) and consider the system of defining relations (14) by extending the system of invariants (12) and (13).

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